

# $SU(3)$ string tension and the presence of vortices

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Lattice simulations are presented showing the expectation of the fluctuation of the Wilson loop solely by elements of the center to fully reproduce the  $SU(3)$  heavy quark potential. The results are stable under smoothing, and point to thick vortices as being responsible for the full  $SU(3)$  tension. An analytic result on the necessary presence of thick vortices for confinement at weak coupling is also presented.

Recently, substantial numerical evidence has been obtained for the vortex picture of confinement [1] - [4]. Here we present simulations extending our  $SU(2)$  results in [1] to the  $SU(3)$  gauge group. We also present an analytical result [5] on the necessity of the presence of thick vortices for maintaining confinement at weak coupling.

Recall that vortices are characterized by multivalued singular gauge transformations  $V(x) \in SU(N)$  with multivaluedness in the center  $Z(N)$ . Vortex configurations have a pure-gauge asymptotic tail given by  $V(x)$  providing the topological characterization of the configurations irrespective of the detailed structure of the core. Such a  $V(x)$ , if extended throughout spacetime, becomes singular on a closed surface  $\mathcal{V}$  of codimension 2 forming the topological obstruction to a single-valued extension. On the lattice, the surface  $\mathcal{V}$  of codimension 2 is regulated to a coclosed set  $\mathcal{V}$  of plaquettes, i.e. a closed loop of dual bonds in  $d = 3$ ; a closed 2-dimensional surface of dual plaquettes in  $d = 4$ , and so on. This represents the core of a thin vortex, each plaquette in  $\mathcal{V}$  carrying flux  $z \in Z(N)$ . Such a thin vortex is suppressed at large  $\beta$  with a cost proportional to the size of  $\mathcal{V}$ .

Thick vortex configurations can be constructed by perturbing the bond variables  $U_b$  in the bound-

ary of each plaquette  $p$  in  $\mathcal{V}$  so as to cancel the flux  $z$  on  $p$ , and distribute it over the neighboring plaquettes forming a thickened core. If  $\mathcal{V}$  is extended enough, the core may be made thick enough, so that each plaquette receives a correspondingly tiny portion of the original flux  $z$  that used to be on each  $p$  in  $\mathcal{V}$ . Long thick vortex configurations may therefore be obtained having  $\text{tr} U_p \sim \text{tr} 1$  for all  $p$  on  $\Lambda$ . Thus they may survive at weak coupling.

Hybrid vortex configurations having a thick and a thin part are also possible. Such hybrid vortices formed by long thick vortices ‘punctured’ by a short (e.g. one-plaquette-long) thin part may then also survive at weak coupling. The short thin part may serve as a ‘tagging’ of a long thick vortex.

Consider two gauge configurations  $\{U_b\}$  and  $\{U'_b\}$  that differ by such a singular gauge transformation  $V(x)$ , and let  $U[C]$  and  $U'[C]$  denote the respective path ordered products around a loop  $C$ . Then  $\text{tr} U'[C] = z \text{tr} U[C]$ , where  $z \neq 1$  is a nontrivial element of the center, whenever  $V$  has obstruction  $\mathcal{V}$  linking with the loop  $C$ ; otherwise,  $z = 1$ . Conversely, changes in the value of  $\text{tr} U[C]$  by elements of the center can be undone by singular gauge transformations on the gauge field configuration linking with the loop  $C$ . This means that vortex configurations are topologically characterized by elements of  $\pi_1(SU(N)/Z(N)) = Z(N)$ . Thus the fluctuation

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in the value of  $\text{tr}U[C]$  by elements of  $Z(N)$  expresses the changes in the number (mod  $N$ ) of vortices linked with the loop over the set of configurations for which it is evaluated.

Motivated by this picture, we separate out the  $Z(N)$  part of the Wilson loop observable by writing  $\arg(\text{tr}U[C]) = \varphi[C] + \frac{2\pi}{N}n[C]$ , where  $-\pi/N < \varphi[C] \leq \pi/N$ , and  $n[C] = 0, 1, \dots, N-1$ . Thus, with  $\eta[C] = \exp(i\frac{2\pi}{N}n[C]) \in Z(N)$ ,

$$W[C] = \langle \text{tr}U[C] \rangle = \langle |\text{tr}U[C]| e^{i\varphi[C]} \eta[C] \rangle \quad (1)$$

$$= \langle |\text{tr}U[C]| \cos(\varphi[C]) \cos(\frac{2\pi}{N}n[C]) \rangle \quad (2)$$

where the last equality follows from the fact that the expectation is real by reflection positivity, and that it is invariant under  $n[C] \rightarrow (N - n[C])$ . We next define

$$W_{Z(N)}[C] = \langle \cos(\frac{2\pi}{N}n[C]) \rangle \quad (3)$$

for the expectation of the  $Z(N)$  part, which, as noted above, gives the response to the fluctuation in the number (mod  $N$ ) of vortices linking with the loop. In the following we compare the string tension extracted from the full Wilson loop  $W[C]$ , eq. (1), to the string tension extracted from  $W_{Z(N)}[C]$ , eq. (3), for  $N = 3$ .

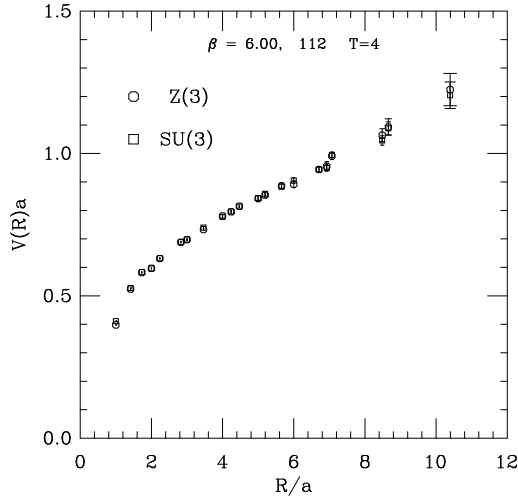


Figure 1. The heavy quark potential at  $\beta = 6.0$  on a set of 112  $12^3 * 16$  lattices.

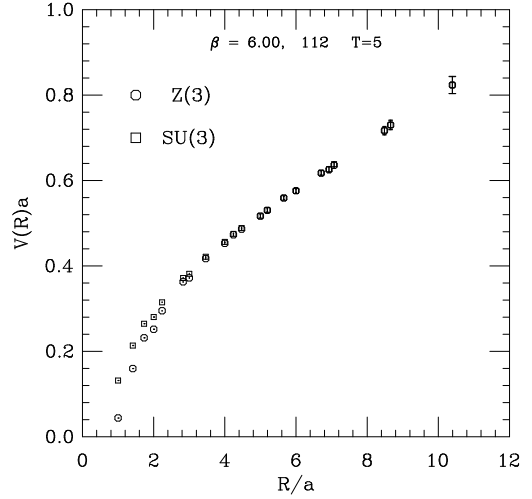


Figure 2. The heavy quark potential at  $\beta = 6.0$  on a set of 112  $12^3 * 16$  lattices from 2 times smoothed lattices.

We worked with the Wilson action at lattice spacings  $a = 0.15$  fm and  $a = 0.10$  fm for  $\beta = 5.8$  and  $\beta = 6.0$ , respectively. This is computed from the string tension assuming that its physical value is 440MeV.

Results are presented in Fig. 1. The agreement between the potential extracted from the full Wilson loop and that from the  $Z(3)$  fluctuation expectation (3) is striking. Note that it includes also the short-distance regime. This is because (3) counts *both* thick and thin vortices, and the thin ones are clearly important at short distances (narrow loops). At longer distances, however, only sufficiently thick vortices can be expected to contribute to the string tension.

To explore this we performed local smoothing on our configurations which removes short distance fluctuations but preserves the long distance physical features. It should be noted that this is in fact essential for checking that topological structure on the lattice is actually well represented. According to well-known results, unless the variation of a lattice configuration over short distance is restricted enough, there is no unambiguous connection to a continuum extrapolation and hence assignment of topology. If the string tension is then really fully reproduced by the vortex fluctuations, the agreement seen in Fig. 1

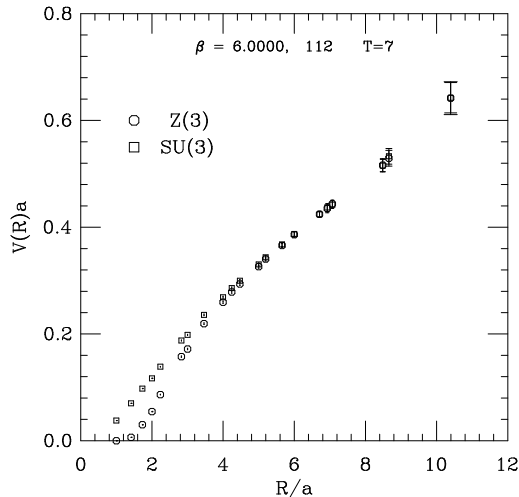


Figure 3. The heavy quark potential at  $\beta = 6.0$  on a set of  $112 \times 12^3 \times 16$  lattices from 6 times smoothed lattices.

should persist at long distances when the potential is measured on the smoothed configurations. This is indeed a very stringent test. We used the smoothing procedure of Ref. [6] applied here to  $SU(3)$ . Results for the potentials on twice smoothed configurations are given in Fig. 2.

We see that the potentials extracted from  $W[C]$  and  $W_{Z(3)}[C]$  now disagree over short distances, but then again merge together with no discernible difference at distances  $R/a > 3$ . This is as expected: smoothing destroys thin vortices but leaves vortices thicker than the smoothing scale unaffected.

Performing successive smoothing steps extends the distance scale over which fluctuations are smoothed; but the asymptotic string tension should not be affected, since, for sufficiently large loops, there is a scale beyond which linked thick vortices are not affected. This is clearly illustrated by comparing Fig. 2 to Fig. 3 which displays the potentials resulting on six times smoothed configurations.

What if, on the other hand, we eliminate all thick vortices linked with the loop, but leave *thin* vortices intact? A very convenient formalism, which explicitly separates thin and thick vortices, recasts the  $SU(N)$  theory in a  $Z(N) \times SU(N)/Z(N)$  form [1]. Thin vortices are then described purely in terms of  $Z(N)$  variables. In

this language it is straightforward to modify the Wilson loop operator by a constraint that forbids its fluctuation by center elements due to linkage with thick vortices. We should then expect the string tension to actually vanish. In fact, we have obtained an analytic proof of this fact. For  $SU(2)$  we show [5] that:

For sufficiently large  $\beta$ , and dimension  $d \geq 3$  the so constrained Wilson loop expectation exhibits perimeter law, i.e. there exist constants  $\alpha$ ,  $\alpha_1(d)$ ,  $\alpha_2(d)$  such that

$$W[C] \geq \alpha \exp(-\alpha_2 e^{-\alpha_1 \beta} |C|) \quad (4)$$

Here  $|C|$  denotes the perimeter length of the loop  $C$ . In other words, the potential indeed becomes nonconfining at weak coupling.

In conclusion, all our results are consistent with a physical picture of locally smooth extended thick vortices occurring over all large scales, and giving rise to the full asymptotic string tension. As mentioned above, individual thick vortices may be ‘tagged’ by the insertion of a short thin segment. We have proposed in the past that such tagging may be used to estimate the contribution of vortices more directly. Ongoing work in this direction will be reported elsewhere.

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